Relative Energy Methods for Models of Compressible Multi-Phase Flows

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Lecture 2: Discontinuous Galerkin Schemes for Compressible Multi-Phase Dynamics

We consider the following one-dimensional model in Lagrangian coordinates

$$\begin{aligned} \partial_t \tau - \partial_x u &= 0 \\ \partial_t u - \partial_x (h'(\tau)) &= -\kappa \partial_{xxx} \tau \end{aligned} \tag{1}$$

where τ is specific volume, u is velocity, $h = h(\tau)$ is an energy density given by a constitutive relation and $\kappa > 0$ is a capillarity constant. For appropriate boundary conditions smooth solutions of (1) conserve the energy functional

$$\int h(\tau) + \frac{\kappa}{2}(\partial_x \tau)^2 + \frac{1}{2}u^2 \, dx.$$

Equation (1) can be seen as a Lagrangian version of a special case of Euler-Korteweg system.

We study the construction of stable numerical methods of Runge-Kutta discontinuous Galerkin type for (1) and describe how the relative energy framework, discussed in the first lecture, can be used for deriving a priori and a posteriori error estimates for such discretisations of (1).

This lecture is a continuation of the lecture which will be delivered on 9 November at the Seminar on Continuum Mechanics, at 15:40 in lecture hall K1.