Aggregation Based Algebraic Multigrid Methods

Geoemtric multigrid methods have had great success in solving the linear systems arising from discretization of partial differential equations, usually on fairly regular grids. The idea is to use computations on coarser grids to improve the convergence of an iterative method for solving the fine grid equations. Algebraic multigrid methods have been devised to deal with problems for which it is not clear how a "coarse grid" should be defined; the problems may not come from a grid-based discretization at all. Based only on the matrix entries, algebraic multigrid methods attempt to define a smaller set of variables and an operator of smaller dimension to play the role of the coarse grid variables and operator in geometric multigrid. A critical part of a multigrid method is the interpolation operator used to move vectors from coarse to fine grids or from spaces of smaller dimension to ones of larger dimension. *Aggregation methods* use the simplest type of interpolation, namely, piecewise constant. Projection from fine to coarse grids is then defined as the transpose of interpolation, and if P is the matrix representing interpolation from a coarse to fine grid and A is the matrix on the fine grid, then $P^T AP$ is the coarse grid matrix.

While piecewise constant interpolation may seem too simple – information from a piecewise linear or bilinear approximation will be lost – it does have some advantages. A big problem for many algebraic multigrid methods is that as problems are transferred to coarser and coarser grids, the matrices become more and more dense. Aggregation based methods maintain better sparsity in the coarse grid matrices and require less setup time. While the multigrid V-cycle usually does not perform as well with aggregation methods, this can be enhanced through an outer conjugate-gradient-like iteration (flexible CG) and an inner cycle that performs extra Krylov iterations on coarse grids when needed. Moreover, analysis by Napov and Notay [1] enables one to analyze the convergence of 2-grid aggregation methods, and further analysis in [3, 2] shows that the 2-grid results carry over to the multigrid setting when a *K-cycle* is used.

Here we will apply this analysis to a finite element discretization of a rotated anisotropic diffusion equation. Such equations are difficult for algebraic multigrid methods because to obtain good performance, the aggregates must be aligned with the direction of anisotropy. We suggest a way to use the analysis in [1] to automate the choice of aggregates in a better way than is done in current codes. We demonstrate the success of the approach in a 2-grid setting for problems with a variety of strengths and directions of anisotropy.

This is joint work with Alan Chen.

References

[1] A. Napov and Y. Notay, *Algebraic analysis of aggregation-based multigrid*, Numer. Lin. Alg. Appl., 18 (2011), pp. 539-564.

[2] A. Napov and Y. Notay, *An algebraic multigrid method with guaranteed convergence rate*, SIAM J. Sci. Comput., 34 (2012), pp. 1079-1109.

[3] Y. Notay and P. Vassilevski, *Recursive Krylov-based multigrid cycles*, Numer. Lin. Alg. Appl., 15 (2008), pp. 473-487.