Very High Order $P_N P_M$ Schemes on Unstructured Meshes for Time–Dependent PDE in Fluid Mechanics

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Abstract

In this talk we present a new unified approach of general $P_N P_M$ schemes on unstructured meshes in two and three space dimensions for the solution of time–dependent partial differential equations arising in fluid mechanics, such as the compressible Euler and Navier–Stokes equations, the classical and relativistic MHD equations or other PDE systems that govern multi–fluid and multi–material flows. The new $P_N P_M$ approach uses piecewise polynomials u_h of degree N to represent the data in each cell. For the computation of fluxes and source terms, another set of piecewise polynomials w_h of degree $M \geq N$ is used, which is computed from the underlying polynomials u_h using a reconstruction or recovery operator. The $P_N P_M$ method contains classical high order finite volume schemes ($N = 0$) and high order discontinuous Galerkin (DG) finite element methods ($N = M$) just as two particular special cases of a more general class of numerical schemes. Our method also uses a novel high order accurate one–step time discretization, based on a local space–time discontinuous Galerkin predictor, which is also able to solve PDE with stiff source terms. We show that our method is asymptotic preserving for a linear model system.

Key words: high order $P_N P_M$ schemes, WENO finite volume schemes, discontinuous Galerkin finite element method, unstructured meshes, ADER approach

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