Maximum-norm a posteriori error estimates for singularly perturbed reaction-diffusion problems

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In the first part of the lectures we consider stationary reaction-diffusion equations of the type

$$\mathcal{L}u := -\varepsilon^2 u_{xx} + cu = f \quad \text{in} \quad (0,1),$$

while the second part ist concerned with its time-dependent analogue

$$\mathcal{M}u := u_t - \varepsilon^2 u_{xx} + cu = f \quad \text{in} \quad (0,1) \times (0,T].$$

Both are equipped with homogeneous Dirichlet boundary conditions and, in case of the time-dependent problem, with appropriate initial conditions. The parameter $\varepsilon > 0$ is small, while the reaction cofficient c is assumed to satisfy $c \geq \gamma^2$ with some positive constant γ.
The efficiency of standard numerical methods deteriorates as the perturbation parameter ε approaches zero. This is because layers form. These are regions where the solution varies rapidely.
In the present talk, bounds for the Green's function associated with the differential operators $\mathcal{L}$ and $\mathcal{M}$ are derived. These bounds are applied to obtain a posteriori error estimators in the maximum norm for difference schemes and for FEM. These estimators are robust with respect to the perturbation parameter. They can be applied to design adaptive mesh-movement algorithms that give numerical methods which converge uniformly with respect to the perturbation parameter ε.
Numerical results will be presented to illustrate the theoretical findings.