Laplacian preconditioning of elliptic operators: localization of the eigenvalues of the discretized problem

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In order to ensure the computational efficiency of Krylov subspace methods used for solving algebraic problems arising in numerical solution of partial differential equation, the discretized problem is typically transformed into a problem that is easier to solve via the given iterative process. Such transformation is historically called preconditioning. Typically one seeks a preconditioner which yields parameter independent bounds for the extreme eigenvalues resulting in convergence bounds based on a single number characteristics, such as condition number of the preconditioned matrix. However, the Krylov subspace methods are strongly nonlinear in the input data and more information about the spectrum is needed in order to capture the actual convergence behavior.

In this talk we focus on second order elliptic PDEs $\nabla \cdot (k(x)\nabla u) = f$ preconditioned by the Laplace operator and analyze the eigenvalues of the resulting preconditioned matrix. Without any assumption on continuity of $k(x)$, we prove the existence of a one-to-one pairing between the eigenvalues of the preconditioned matrix and the intervals determined by the images under $k(x)$ of the supports of the FE nodal basis functions. As a consequence, we can show that the nodal values of $k(x)$ yield accurate approximations of the eigenvalues.