On direct multilevel solvers induced by AMG

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Abstract

We consider an algebraic multigrid (AMG) scheme for the direct solution of complex-valued square linear systems based on a recursive 2×2 block partitioning of the coefficient matrix and study the optimal choices of its components. In particular, we complement existing results that characterize optimal choices for nonsymmetric-cycle methods [1, 2] by analyzing the spectral behavior of the symmetrized variant. We analyze the error propagation operator of the two-level symmetric-cycle method by calculating its invariant subspaces and its nonzero eigenvalues that govern the behavior of the error after a single cycle. We show that the error propagation operator can be studied separately for pairs of modes, working as bases of the invariant subspaces. The main result is an explicit choice of smoothing parameters that makes all the pairs of modes respond identically, forcing the nontrivial eigenvalues of the error propagation operator to collapse to a single, a priori known value, i.e., we give an explicit choice for the smoothing parameters so that the nonzero part of the spectrum of the error propagation operator collapses to a single, a priori known point on the real line. As a consequence, we give a closed-form formula for the inverse of a general square matrix that, to the best of our knowledge, is new in the literature. Additionally, the framework provides a clear, self-contained description of an ideal AMG W-cycle, offering a concrete target for the design of related schemes. We illustrate the theory with direct applications to general matrices and with analyses of representative matrices arising in numerical methods.

First part of the talk will be devoted to recalling/introduction of geometric multigrid and the underlying ideas (target audience are MSc. students of the department). This talk will have a continuation on the next week seminar by Pablo.

References

[1] S. MacLachlan, T. Manteuffel, and S. F. McCormick. Adaptive reduction-based AMG. *Numerical Linear Algebra with Applications*, volume 13(8): 599–620, 2006.

[2] J. P. Lucero Lorca. Towards a multigrid preconditioner interpretation of Hierarchical Poincaré-Steklov solvers. *arXiv:2511.00735*, 2026.