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Numerical approximation of the spectrum of self-adjoint operators and operator preconditioning

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Abstract

We consider operator preconditioning $\mathcal{B}^{-1}\mathcal{A}$, which is employed in the numerical solution of boundary value problems. Here, the self-adjoint operators $\mathcal{A}, \mathcal{B} : H_0^1(\Omega) \to H^{-1}(\Omega)$ are the standard integral/functional representations of the partial differential operators $-\nabla \cdot (k(x)\nabla u)$ and $-\nabla \cdot (g(x)\nabla u)$, respectively, and the scalar coefficient functions $k(x)$ and $g(x)$ are assumed to be continuous throughout the closure of the solution domain. The function $g(x)$ is also assumed to be uniformly positive. When the discretized problem, with the preconditioned operator $\mathcal{B}_n^{-1}\mathcal{A}_n$, is solved with Krylov subspace methods, the convergence behavior depends on the distribution of the eigenvalues. Therefore it is crucial to understand how the eigenvalues of $\mathcal{B}_n^{-1}\mathcal{A}_n$ are related to the spectrum of $\mathcal{B}^{-1}\mathcal{A}$. Following the path started in the two recent papers published in SIAM J. Numer. Anal. [57 (2019), pp. 1369-1394 and 58 (2020), pp. 2193-2211], the first part of the talk addresses the open question concerning the distribution of the eigenvalues of $\mathcal{B}_n^{-1}\mathcal{A}_n$ formulated at the end of the second paper.

The second part generalizes some of the results to bounded and self-adjoint operators $\mathcal{A}, \mathcal{B} : V \to V^{\#}$, where $V^{\#}$ denotes the dual of V. More specifically, provided that $\mathcal{B}$ is coercive and that the standard Galerkin discretization approximation properties hold, we prove that the whole spectrum of $\mathcal{B}^{-1}\mathcal{A} : V \to V$ is approximated to an arbitrary accuracy by the eigenvalues of its finite dimensional discretization $\mathcal{B}_n^{-1}\mathcal{A}_n$.

The presented spectral approximation problem includes the continuous part of the spectrum and it differs from the eigenvalue problem studied in the classical PDE literature which addresses compact (solution) operators.