Acceleration of FFT-based homogenisation by low-rank tensors approximations

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Abstract. The purpose of computational homogenization is to describe the macroscopic response of heterogeneous materials based on their microstructure properties. Evaluation of the homogenised properties with high accuracy requires detailed knowledge of materials' microstructure. Unfortunately, this knowledge comes hand in hand with high memory and time requirements.

To find a solution we use the method based on the fast Fourier transform (FFT), which has been introduced in 1994 by Moulinec and Suquet [1] and later explained as the Fourier-Galerkin method by Vondřejc, Zeman and Marek in [2]. FFT-based methods have turned out to be effective computational approaches for numerical homogenisation of periodic media. Its computational efficiency benefits from an efficient FFT algorithm as well as a favorable condition number of the ensuing system of linear equations.

We accelerated this method by low-rank tensor approximation techniques for both the input data and the solution. The low-rank tensors approximate large multidimensional tensors by a truncated sum of one-dimensional vectors. Hackbusch explained the whole concept on a variety of low-rank tensor formats in [3]. We have tested canonical, Tucker and tensor train [4] formats.

Although this data compression reduces computational requirements significantly, it also introduces additional errors. Thus, we had to adjust the original method to the low-rank tensor format, with emphasis on the reformulation of the solver. Our numerical results demonstrate that the low-rank solver offers at least a thousandfold reduction in the memory requirements and a tenfold reduction in the simulation time while generating sufficiently accurate solutions for industrial-scale applications.

References

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