29th Colloquium Lecture, School of Mathematics
Faculty of Mathematics and Physics

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On the zeros of (rational) harmonic functions

Abstract

In this talk we will discuss results about the zeros of harmonic functions, i.e., functions **f** in the complex plane which have a local decomposition of the form **f(z)=h(z)+g(z)***, where **h** and **g** are analytic. Motivated by a problem in matrix theory we will first focus on rational harmonic functions of the form **f(z)=r(z)-z***. Using results about fixed points from the area of complex dynamics we will derive bounds on the number of zeros of such functions which depend on the degrees of the numerator and denominator polynomials of **r**. We will then discuss how a constant shift, i.e., forming **f(z)-η**, affects these zeros. This is of interest in gravitational lensing theory, where **f** represents a gravitational point-mass, and **η** corresponds to a light source. After explaining how the results can be extended to the general case **f(z)=h(z)+g(z)***, we will discuss the recently developed **harmonic Newton method** for numerically computing the zeros of harmonic functions.

References

[1] J. Liesen and J. Zur, The maximum number of zeros of $r(z)-\conj{z}$ revisited, Comput. Methods Funct. Theory, 18 (2018), pp. 463-472.

[2] J. Liesen and J. Zur, How constant shifts affect the zeros of certain rational harmonic functions, Comput. Methods Funct. Theory, 18 (2018), pp. 583–607.

[3] O. S{\'e}te and J. Zur, A Newton method for harmonic mappings in the plane, to appear in IMA J. Numer. Anal., 2019.