Augmented Krylov subspace methods for well- and ill-posed problems

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Abstract. Krylov subspace methods have been quite successful in treating linear systems of the form $\mathbf{A}(\mathbf{x}_0 + \mathbf{t}) = \mathbf{b}$ in the case that $\mathbf{A}$ is large and sparse or only available as a procedure encoding the action of $\mathbf{A}$ applied to vectors. They work by selecting approximations to the error $\mathbf{t}$ from a Krylov subspace, usually of the form $\widehat{\mathbf{t}} \in \mathcal{K}_j(\mathbf{A}, \mathbf{r}_0) = \operatorname{span}\left\{\mathbf{r}_0, \mathbf{A}\mathbf{r}_0, \mathbf{A}^2\mathbf{r}_0, \ldots, \mathbf{A}^{j-1}\mathbf{r}_0\right\}$, $\mathbf{r}_0 = \mathbf{b} - \mathbf{A}\mathbf{x}_0$. However, it is often the case that one posseses a useful subspace that one would like to include in the reconstruction process, while still building a Krylov subspace, so that the approximation $\mathbf{t} \approx \widehat{\mathbf{t}} \in \mathcal{U} + \mathcal{K}_j$ where $\mathcal{U}$ is a fixed space known apriori and $\mathcal{K}_j$ is some Krylov subspace which is generated iteratively.

The class of methods which accomplish this are often called recycled or augmented Krylov subspace methods. Many different variants have been proposed both for the treatment of well- and ill-posed problems. In this talk, we will give a general overview of these methods and how they work. We show that all such methods can be described in a common framework which is quite flexible (moreso than previously proposed versions of the same). We demonstrate that this high-level understanding allows us to quite easily propose new augmented iterative methods which can be tailored to the (mathematical or computational) needs of the application. We will walk through a couple of examples to demonstrate the ease with which one can do this using the framework. We will finish with some numerical experiments, demonstrating the performance of the methods we have just proposed.